

Problem Set #6

Quantum Error Correction

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Problem #1. Teleportation of the controlled-phase gate

Let U be the gate

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}. \quad (1)$$

- a) Find UPU^\dagger for $P = X \otimes I, I \otimes X, Z \otimes I,$ and $I \otimes Z$.
- b) Find a *two-qubit* ancilla state and a circuit involving two data qubits and the ancilla consisting of Clifford group gates, Pauli basis measurements, and classical feed-forward, such that the output of the circuit is $U|\psi\rangle$. ($|\psi\rangle$ is the input state of the two data qubits.)

Problem #2. Correctness with weaker versions of fault tolerance

For this problem, we will consider a fault-tolerant protocol for a QECC capable of correcting t errors.

- a) Suppose we consider weaker versions of the ECRP and the GPP. The weak ECRP has a $2s$ -filter on the right-hand-side for the output instead of an s -filter when there are s faults in the EC gadget, with $2s \leq t$. The weak GPP (for a single-block gate) has an $(r + 2s)$ -filter instead of an $(r + s)$ -filter for the output on the right-hand-side when the input passes an r -filter and there are s faults in the circuit, with $r + 2s \leq t$. The correctness conditions are unchanged. For a single-block gate exRec, how many faults can we allow within the exRec and still guarantee that the exRec is correct?
- b) Consider the weak version of the ECRP and a very weak version of the GPP, with a $(2r + 2s)$ -filter for the output on the right-hand-side instead of a $(r + 2s)$ -filter, for $2r + 2s \leq t$. With these properties, how many faults can we allow within the exRec and still guarantee that the exRec is correct?