Homework set 5, PHY 790, OPTIONAL (extra credit) due whenever

18. (a) In nonrelativistic physics, shouldn't a box of dilute gas be cooler at the top because its molecules slow down as they rise? What is the flaw in this reasoning?
(b) What would $\Delta T/T$ be for a box of gas in a gravitational field, if you took into account the special relativistic relationship $E = \sqrt{\vec{p}^2 + m^2}$ but ignored all other relativistic effects (i.e. if you took the Hamiltonian to be $H = \sqrt{\vec{p}^2 + m^2} + mgh$)?

19. (a) Verify the formula quoted in class,

$$t = \int \frac{dl}{e^\Phi} + \int \frac{g_{0k}\,dx^k}{|g_{00}|} \qquad (e^\Phi = \sqrt{|g_{00}|})$$

for the "time of flight" that enters into Fermat's principle for light rays.
(b) Specializing this formula to a Schwarzschild spacetime, show that Fermat's principle leads to the same equation for a photon orbit as we found in class. (Hint: you can use the φ-independence to reduce the variational equation $\delta t = 0$ to first order.)

20. Consider three vectors a, b, c, and two covectors d, e such that

$$(d,a) = 2, (d,b) = 3, (d,c) = 4, \ (e,a) = -1, (e,b) = 2, (e,c) = -3$$

Let

$$T^{ab} = a^a b^b - a^b b^a + c^a c^b$$

Simplify the following expressions: (a) $T^{ab} d_a e_b$; (b) $T^{ab} e_a d_b$; (c) $T^{ab} e_b$;