Quantum effects across dynamical horizons

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Intuitive Idea and Summary of the Main Result

We present a description of the Hawking effect as a local process using the tunneling picture. In this framework, the Hawking particle tunnels through the horizon on a complex path. Following the example of the black hole, the tunneling picture allows to generalise the Hawking effect to all dynamical trapping horizons. The idea is to identify the classically forbidden direction of horizon crossing which differs from setup to setup. Although classically not allowed, this direction might serve as a tunneling path for the Hawking effect. Pursuing this logic further, we find that the Hawking effect manifests itself as an absorption for past outer horizons (e.g.white holes) and future inner horizons (e.g. inner horizons of black holes) and is confirmed to be an emission for future outer horizons (e.g. black hole horizon) and past inner horizons (e.g. Hubble sphere).

Dynamical Horizons

Characterisation of different regions in space-time by the null expansions θ^- and θ^+ along ingoing and outgoing null rays l^- and l^+. Division into three regions: normal ($\theta^+\theta^- < 0$), trapped ($\theta^+\theta^- > 0$) and horizons (or marginally trapped surfaces with either $\theta^+ = 0$ or $\theta^- = 0$).

- **Future horizons** ($\theta^+ = 0$)
  - outer horizon ($\mathcal{L}_-\theta^+|_{R_H} < 0$): black hole horizon, ingoing: allowed, outgoing: forbidden.
  - inner horizon ($\mathcal{L}_-\theta^+|_{R_H} > 0$): big crunch horizon, ingoing: allowed, outgoing: forbidden.
- **Past horizons** ($\theta^- = 0$)
  - outer horizon ($\mathcal{L}_+\theta^-|_{R_H} < 0$): white hole horizon, ingoing: forbidden, outgoing: allowed.
  - inner horizon ($\mathcal{L}_+\theta^-|_{R_H} > 0$): big bang horizon, ingoing: forbidden, outgoing: allowed.

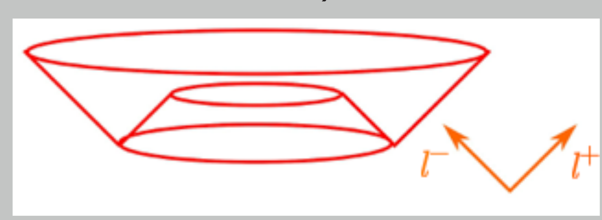

Figure 1: Normal region: the initial surface area decreases along l^- and increases along l^+.

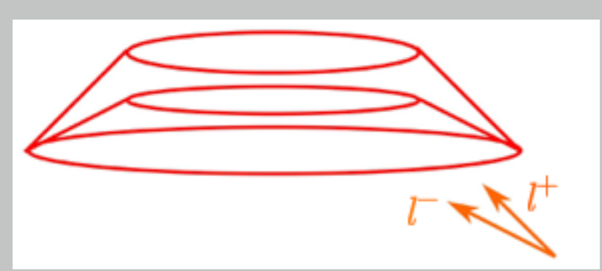

Figure 2: Trapped region: the initial surface area decreases along both l^- and l^+. For anti-trapped we see an increase in both directions.

Tunneling Picture

- Assumptions
  - local neighbourhood near the horizon R_H given by the Bardeen metric:
$$g = -e^{2\psi(\xi,R)}C(R)\mathrm{d}\xi \otimes \mathrm{d}\xi \pm 2e^{\psi(\xi,R)}\mathrm{d}\xi \otimes \mathrm{d}R + R^2\mathrm{d}^2\Omega \quad (1)$$
  - the horizon can have an adiabatic time-evolution
  - spherical symmetry is maintained

  $\Rightarrow$ Quantum correlations across the horizon can be approximated by the tunneling rate in the WKB framework: $\langle\phi(r_<)\phi(r_>)\rangle \to \Gamma_{\mathrm{WKB}}$
- Invariant quantities: using the Kodama vector $K = (e^{-\psi(\xi,R)}, \vec{0})$ to define
  - energy for a particle in motion: $\omega = -K^\mu\partial_\mu S_0 = -e^{-\psi(\xi,R)}\partial_\xi S_0$
  - surface gravity: $\kappa_H = \frac{1}{2}\partial_R C(R)|_{R=R_H}$
- Hamilton-Jacobi method
  - scalar field given by $(\Box - m^2 + i\varepsilon)\phi = 0$ (Feynman prescription)
  - WKB: $\phi = exp(i/\hbar S_0 + S_1 + \mathcal{O}(\hbar))$ with S_0 the classical action
  - Hamilton-Jacobi equation: $g^{-1}(\mathrm{d}S_0, \mathrm{d}S_0) + m^2 - i\varepsilon = 0$ is solved by $S_0 = \int \partial_\xi S_0 \mathrm{d}\xi + \int \partial_R S_0 \mathrm{d}R$ assuming horizon crossing happens on almost null paths ($m \approx 0$)

  $\Rightarrow$ Tunneling probability $\Gamma_{\mathrm{WKB}} \propto \exp(-2Im(S_0)/\hbar)$

  $\Rightarrow$ Non-trivial pole in S_0 induces a non-zero principal value in $k = \partial_R S_0$ at $R = R_H$ for the classically forbidden direction:
$$k_{\mathrm{outgoing}} = \frac{2\omega}{C(R)} + \frac{i\varepsilon}{2\omega}, \text{ (BH example)} \quad (2)$$
  $\Rightarrow$ Non-trivial imaginary part: $\mathbf{Im}(S_0) = \pi\omega/\kappa_H$

  $\Rightarrow$ WKB Tunneling rate: $\Gamma_{\mathrm{WKB}} \propto \exp(-2\pi|\omega|/\hbar\kappa_H)$

  $\Rightarrow$ Possibility to assign a temperature T_H to each horizon

Thermality of Quantum Effects

Principle: The Hawking effect describes horizon crossings on imaginary paths which are classically forbidden but quantum-theoretically allowed.
$\Rightarrow$ Sufficient and necessary to perceive a Hawking effect: $\mathbf{Im}(S_0) > 0$.
To assign a temperature one needs to look careful at the process itself. Consider the production of a Hawking pair: particle with $\omega > 0$ and partner particle with $\omega < 0$.

- Comparison of Γ_{WKB} with the Boltzmann distribution $\rho_B(E) = e^{-E/T}$:
  - **Emission:** Counting more particles with $\omega > 0$ in the normal region $\leftrightarrow$ tunneling of partner with $\omega < 0$ in the trapped region (FOTH/PITH)
    Probability of having a particle more at fixed E is suppressed
    $\Rightarrow P_{\mathrm{em}} = e^{-E/T}P_{\mathrm{abs}}$ with $E > 0 \Rightarrow 2\pi T = \kappa_H > 0$
- For inner horizons $\kappa_H < 0$ but a compensating sign arises from the changed direction of the tunneling path
  - **Absorption:** Counting less particles with $\omega > 0$ in the normal region $\leftrightarrow$ tunneling of partner with $\omega < 0$ in the normal region (FITH/POTH)
    Probability of having a particle less at fixed E is suppressed
    $\Rightarrow P_{\mathrm{ab}} = e^{-E/T}P_{\mathrm{em}}$ with $E > 0 \Rightarrow T = \kappa_H/2\pi > 0$

Note: The legitimation for a thermal interpretation roots in the QFT description. Whenever a certain horizon is present the state will acquire thermal properties (KMS state). The dynamics of the horizon create additional terms in $\mathbf{Im}(S_0)$ which depart from an equilibrium configuration.

Illustrations

Illustration of the Hawking process: Blue arrows denote the classically allowed direction, while red ones the forbidden ones. Dashed blue paths symbolise complexified (tunneling) paths of the Hawking effect.

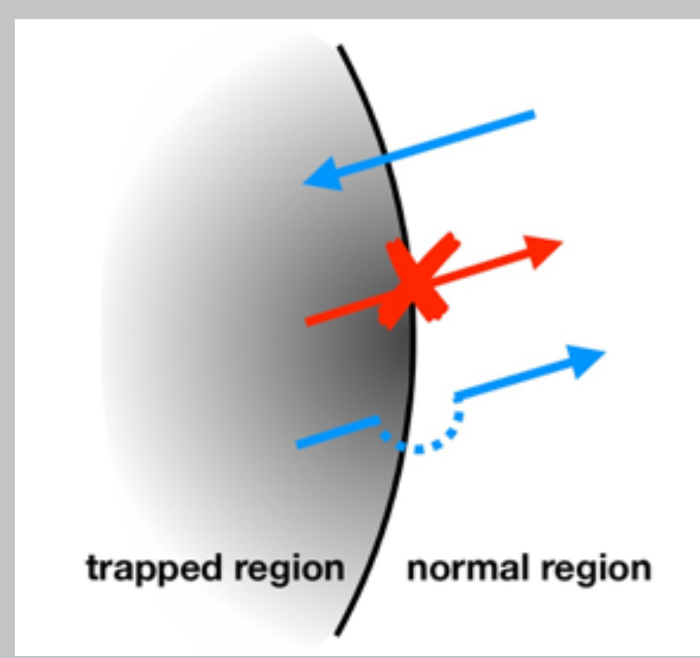

Figure 3: FOTH (black hole)

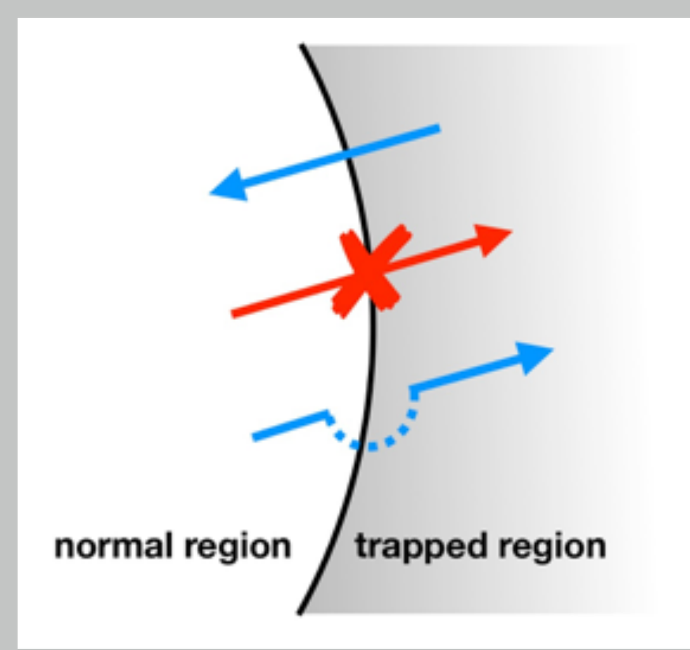

Figure 5: FITH (contracting cosmology)

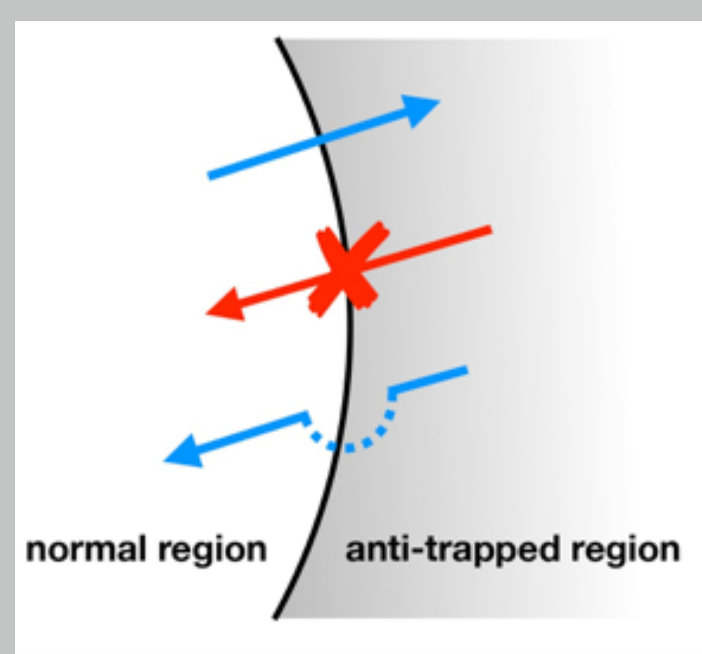

Figure 4: PITH (expanding cosmology)

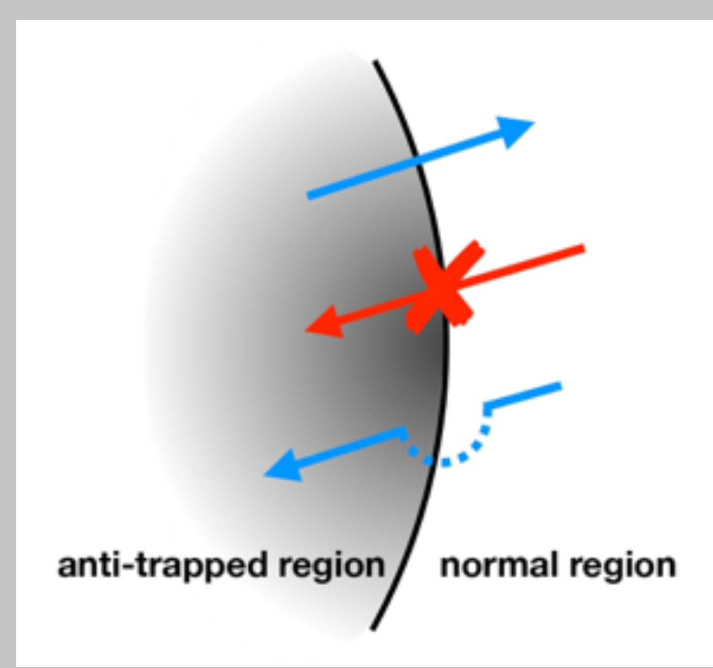

Figure 6: POTH (white hole)

Conclusion

- The tunneling picture allows to describe the Hawking effect as a local phenomenon which is generalisable for dynamical horizons
- All dynamical horizons are subjected to a Hawking process with positive temperature $T = \kappa_H/2\pi$.
- Quantum effects across horizons can be unified within the tunneling picture

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