Parity Horizons, Black Holes, and Chronology Protection in Shape Dynamics

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Shape Dynamics:

Shape Dynamics describes gravity as the time-evolution of the shape of space.

Geometry vs. Shape:

Geometry:

The (Riemannian) geometry of space can be completely described by a Euclidean-signature metric tensor $q_{ij}(x)$ up to diffeomorphisms:

$$x^i \to \tilde{x}^i(x)$$

$$\tilde{q}_{kl}(\tilde{x}) = q_{ij}(x)\frac{\partial x^i}{\partial \tilde{x}^k}\frac{\partial x^j}{\partial \tilde{x}^l}$$

Shape:

The *shape* of space can be described by a Euclidean-signature metric tensor $q_{ij}(x)$ up to diffeomorphisms *and spatial Weyl transformations*:

$$q_{ij}(x) \to e^{4\phi(x)}q_{ij}(x)$$

- Weyl transformations can be thought of as a local rescaling of the geometry.
- Shape dynamics is invariant under spatial diffeomorphisms and spatial Weyl transformations:
  It is a theory of the shape of space.

Gauge Symmetries and Constraints

General Relativity:

- General relativity is invariant under space-time diffeomorphisms.
- In the Canonical formalism, this is split into two parts.

Spatial diffeomorphism generated by the momentum constraint:

$$\mathcal{H}_a(x) = \pi^a_{b;a}$$

Refoliations generated by the Hamiltonian constraint:

$$\mathcal{H}(x) = \frac{G_{abcd}\pi^{ab}\pi^{cd}}{\sqrt{q}} - R\sqrt{q}$$

Shape Dynamics:

Shape Dynamics is invariant under spatial diffeomorphisms generated by the same momentum constraint

$$\mathcal{H}_a(x) = \pi^a_{b;a}$$

as well as spatial Weyl tranformations generated by a Weyl constraint:

$$\mathcal{D}(x) = q_{ij}\pi^{ij}$$

Where π^{ij} is canonically conjugate to q_{ij}:

$$\{q_{ij}(x), \pi^{kl}(x')\} = \delta_i^{(k}\delta_j^{l)}\delta(x - x').$$

Second-Class Constraints

Unlike general relativity, shape dynamics also has a system of second-class constraints.

$$e^{-4\phi}(\bar{\nabla}^2 N + 2\bar{q}^{ij}\phi_{,i}N_{,j}) - e^{-6\phi}N\bar{G}_{ijkl}\frac{\bar{\pi}^{ij}\bar{\pi}^{kl}}{|\bar{q}|} \approx 0$$

$$\sqrt{\bar{q}}\left(8\bar{\nabla}^2\phi - \bar{R}\phi\right) + \frac{\bar{\pi}_{ij}\bar{\pi}^{ij} - \bar{\pi}^2}{\sqrt{\bar{q}}}\phi^{-7} \approx 0.$$

Where N is a lapse function, $q_{ij} = e^{4\phi}\bar{q}_{ij}$, $\pi^{ij} = e^{-4\phi}\bar{\pi}^{ij}$ and $\bar{G}_{ijkl} = \frac{1}{2}(\bar{q}_{ik}\bar{q}_{jl} + \bar{q}_{il}\bar{q}_{jk}) - \bar{q}_{ij}\bar{q}_{kl}$.

These constraints do not weakly commute with other constraints and must be solved in order to find solutions.

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- Constraint Algebra is a Lie Algebra $\Longrightarrow$
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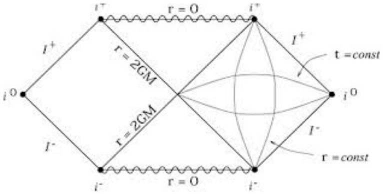
- Time plays a clearer, more physical role in the theory.
- All of these features could eventually facilitate canonical quantization of gravity.

Two Problems with General Relativity

GR Predicts its own Demise:

Black hole solutions of general relativity collapse to physical singularities where the theory breaks down.



GR predicts CTCs:

Many solutions of general relativity contain closed time-like curves, allowing observers to revisit events in their past.

Shape Dynamic Black Holes

Shape Dynamic black holes are physically different from general relativistic black holes:

Example: Spherically Symmetric Black Hole.

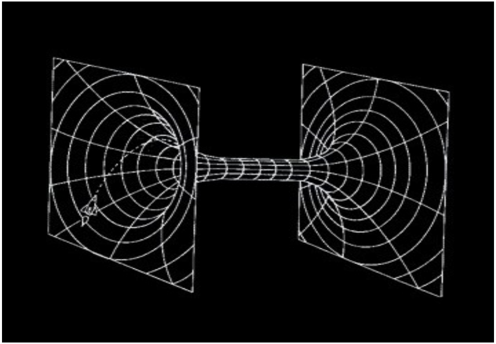
$$ds^2 = -\left(\frac{1-\frac{m}{2r}}{1+\frac{m}{2r}}\right)^2 dt^2 + \left(1+\frac{m}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right))$$

(Gomes, 2013. arXiv:1305.0310 [gr-qc])

Invariant under a combination of $r \to m^2/4r$ and $t \to -t$.

Shape Dynamic Black Holes are *Wormholes*

This solution represents a *wormhole*.



$\implies$ No singularity at $r = 0$!

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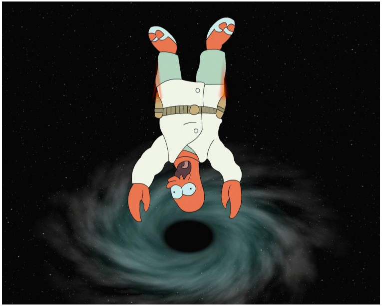

- An infalling observer would take infinite proper time to reach $r = 0$. This is *physically different* than the Schwarzschild spacetime!

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- This makes the wormhole character of the solution obvious.
- This transformation also shares many features of a parity transformation.
- The spherically symmetric shape dynamic black hole is invariant under a combination of this "parity" and time-reversal— what happens if we add in charge?

Coupling Shape Dynamics to the Electromagnetic Field

The Hamiltonian for electromagnetism is given by:

$$\mathcal{H}_{\text{EM}} = 2\sqrt{\bar{q}}\left(-A_{[i,j]}A_{[k,l]}\bar{q}^{ak}\bar{q}^{jl} + \bar{E}^i\bar{E}^j\bar{q}_{ij}\right)$$

From which we obtain the coupled second-class constraints:

$$\sqrt{\bar{q}}\left(8\bar{\nabla}^2\phi - \bar{R}\phi\right) + \frac{\bar{\pi}_{ij}\bar{\pi}^{ij}}{\sqrt{\bar{q}}}\phi^{-7} + \mathcal{H}_{\text{EM}} \approx 0$$

$$e^{-4\phi}(\bar{\nabla}^2 N + 2\bar{q}^{ij}\phi_{,i}N_{,j}) - e^{-6\phi}N\left(\bar{G}_{ijkl}\frac{\bar{\pi}^{ij}\bar{\pi}^{kl}}{|\bar{q}|} + \frac{\mathcal{H}_{\text{EM}}}{\sqrt{\bar{q}}}\right) \approx 0.$$

A Charged Shape Dynamic Black Hole

The second-class constraints can be solved exactly if we assume the conformal initial data:

$$\bar{q}_{ij} = \eta_{ij} \qquad \bar{A}_i = 0$$
$$\bar{\pi}^{ij} = 0 \qquad \bar{E}^i = -\delta^i_r \left(\frac{Q}{r^2}\right)$$

Where η_{ij} is the flat spatial metric in spherical coordinates and $\bar{E}^i = e^{6\phi} E^i$ is the spherically symmetric electric field in this background.

A Charged Shape Dynamic Black Hole

Since the conformal initial data is written in terms of the flat spatial metric η_{ij}, the scalar curvature R vanishes, and the coupled Lichnerowicz-York constraint

$$\sqrt{\bar{q}}\left(8\bar{\nabla}^2\phi - \bar{R}\phi\right) + \frac{\bar{\pi}_{ij}\bar{\pi}^{ij} - \bar{\pi}^2}{\sqrt{\bar{q}}}\phi^{-7} \approx 0.$$

reduces to the simple expression:

$$8\Omega^3\bar{\nabla}^2\Omega + \frac{\mathcal{H}_{\text{EM}}}{\sqrt{\bar{q}}} = 0.$$

Where $\Omega = e^{\phi}$. Writing $\frac{\mathcal{H}_{\text{EM}}}{\sqrt{\bar{q}}}$ in terms of Q and r gives:

$$8\Omega^3\left(\Omega'' + \frac{2}{r}\Omega'\right) + \frac{2Q^2}{r^4} = 0$$

Where primes denote differentiation with respect to r.

A Charged Shape Dynamic Black Hole

The last equation is difficult to solve in its present form, but it can be simplified by making the substitution $\Omega^2 = \psi$, which yields:

$$-2\left(\psi'\right)^2 + 4\psi\psi'' + \frac{8}{r}\psi\psi' + \frac{2Q^2}{r^4} = 0. \qquad (*)$$

Now this equation can be solved by making a Laurent series ansatz:

$$\psi = \sum_{n=0}^{\infty} c_n r^{-n}.$$

Now the derivatives of ψ can be easily calculated and inserted back into $(*)$ to yield the infinite double sum:

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} c_m c_n \left[-2mn + 4n(n+1) - 8n\right] r^{-(2+m+n)} = -\frac{2Q}{r^4}.$$

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- Imposing the boundary conditions $c_0 = 1$, $c_1 = m$, this equation can be solved for c_2 in terms of the mass m and electric charge Q:

$$c_2 = \frac{m^2 - Q^2}{4}.$$

A Charged Shape Dynamic Black Hole

- With all of the constants in our Laurent series ansatz fixed, we are left with:

$$\psi = 1 + \frac{m}{r} + \frac{m^2 - Q^2}{4r^2} \quad \Longrightarrow \quad \Omega = \left(1 + \frac{m}{r} + \frac{m^2 - Q^2}{4r^2}\right)^{1/2}.$$

- Putting this result back into the coupled lapse-fixing equation gives the second order, ODE

$$\Omega^4 \left(N'' + 2\left(\frac{1}{r} + \frac{\Omega'}{\Omega}\right) N'\right) - \frac{Q^2}{r^4} N = 0$$

- Which with ordinary asymptotically flat boundary conditions has the unique solution

$$N = \frac{1 - \frac{m^2 - Q^2}{4r^2}}{1 + \frac{m}{r} + \frac{m^2 - Q^2}{4r^2}}.$$

A Charged Shape Dynamic Black Hole

The solutions of the second class constraints allow us to reconstruct a Lorentzian line-element:

$$ds^2 = -N^2 dt^2 + e^{4\phi}\left(dr^2 + r^2\left(d\theta^2 + \sin^2(\theta) d\phi^2\right)\right)$$

Where we have chosen the shift vector to be zero in accordance with spherical symmetry.

The physical electric field after conformal rescaling becomes:

$$E^i = e^{-6\phi}\bar{E}^i = -\delta^i_r\left(\frac{Q}{r^2}\right)\left[\left(1+\frac{m}{2r}\right)^2 - \left(\frac{Q}{2r}\right)^2\right]^{-3}$$

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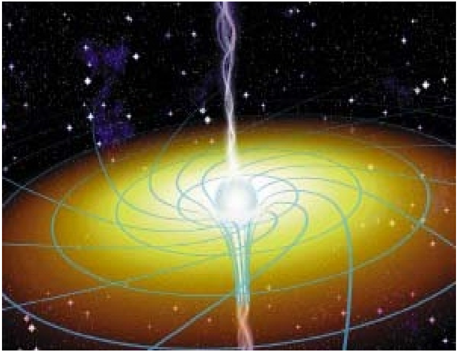
- The lapse is invariant under charge-conjugation and changes sign under parity and time-reversal.
- The electric field is invariant under time-reversal and changes sign under parity and charge-conjugation...

The Solution is CPT Invariant!

Both the gravitational field and the electric field are invariant under CPT transformations.

Axisymmetric Solutions and Rotating Black Holes



H. Gomes, G. Herczeg arXiv:1310.6095 [gr-qc]

The Stationary, Axisymmetric Line Element

We begin our consideration of rotating black holes by analyzing the stationary, axisymmetric line element:

$$ds^2 = -(N^2 - \Omega\Psi\xi^2)dt^2 + \Omega[(dx^1)^2 + (dx^2)^2 + \Psi d\phi^2] + 2\Omega\Psi\xi d\phi dt$$

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Generic local equivalence of GR and shape dynamics
$\implies$ most general local form of the shape dynamics solution.

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where $(\mathcal{L}_\xi q)_{ij}$ denotes the Lie derivative of the spatial metric along the shift vector. Using stationarity and axisymmetry, the trace of this equation becomes:

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Which shows that the stationary, axisymmetric line element satisfies the Weyl constraint.

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- Now that we have established this boring lemma, we can use it for exciting things!
  - We can always express a stationary, axisymmetric general relativistic black hole in a form that satisfies the Weyl constraint.
  - This should give us a local expression for the corresponding solution of shape dynamics.
- In the Boyer-Lindquist coordinates, the Kerr metric takes the form:

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\Sigma}\left((r_{\text{BL}}^2 + a^2)d\phi - adt\right)^2 + \frac{\Sigma}{\Delta}dr_{\text{BL}}^2 + \Sigma d\theta^2$$

where

$$\Delta = r_{\text{BL}}^2 - 2mr_{\text{BL}} + a^2, \quad \Sigma = r_{\text{BL}}^2 + a^2\cos^2\theta.$$

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In these coordinates, the line element reads:

$$ds^2 = -\lambda^{-1}(dt - \omega d\phi)^2 + \lambda[m^2 e^{2\gamma}(d\mu^2 + d\theta^2) + s^2 d\phi^2]$$

where

$$\begin{aligned} s &= mp\sinh\mu\sin\theta \\ e^{2\gamma} &= p^2\cosh^2\mu + q^2\cos^2\theta - 1 \\ \omega &= e^{-2\gamma}\left[2a\sin^2\theta(p\cosh\mu + 1)\right] \\ \lambda &= e^{-2\gamma}\left[(p\cosh\mu + 1)^2 + q^2\cos^2\theta\right] \end{aligned}$$

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- Since the determinant of the space-time metric can be written $\sqrt{-g} = N\sqrt{q}$, the space-time metric is degenerate on the horizon where the lapse vanishes.
- Analysis of conformal-diffeo invariants constructed from the cotton tensor shows no physical singularities on the horizon.

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    - It is free of physical singularities.
    - Unlike the Kerr solution, it is free of inner horizons and closed time-like curves. More on this coming up...

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- Further study is needed to determine whether there is a deep reason for this coincidence.
- Now we will shift gears to discuss how similar “parity horizons” arise in solutions of shape dynamics which are not black holes.

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- and $\mathbb{P}$ changes the sign of the lapse, i.e. $\mathbb{P}[N] = -N$

Then we call $\mathbb{P}$ a parity and $\mathcal{S}_0$ a parity horizon.

We will see that not only the event horizons of black holes, but other types of horizons become parity horizons in shape dynamics.

Rindler Space

The Rindler Chart over Minkowski space represents flat space-time as seen by a congruence of uniformly accelerating observers.

In order to construct the Rindler chart, one can begin with Cartesian coordinates over Minkowski spacetime. The line element is simply:

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2.$$

If one then introduces the coordinate transformation

$$t = \frac{1}{\kappa}\tanh^{-1}\left(\frac{T}{X}\right), \; x = \sqrt{X^2 - T^2}, \; y = Y, \; z = Z$$

one obtains the Rindler chart with the line element

$$ds^2 = -\kappa^2 x^2 dt^2 + dx^2 + dy^2 + dz^2.$$

Rindler Space in Shape Dynamics

- In shape dynamics, Rindler Space can be derived by considering the flat initial data $q_{ij} = \delta_{ij}$, $\pi^{ij} = 0$, and choosing the gauge $\phi = 0 = \xi^i$.
- In this simple case, the lapse-fixing equation reduces to Laplace's equation, $\nabla^2 N = 0$. The general solution of the lapse-fixing equation for this initial data is now trivially given by the harmonic functions.

Asymptotically flat BCs:

$$N|_{r\,\to\,\infty} = 1, \;\; \frac{\partial N}{\partial r}\Big|_{x=0} \sim \mathcal{O}(r^{-2})$$

$$\Longrightarrow \text{ Minkowski:}$$

$$N = 1.$$

Horizon BCs:

$$N|_{x\,=\,0} = 0, \;\; \frac{\partial N}{\partial x}\Big|_{x=0} = \kappa$$

$$\Longrightarrow \text{ Rindler:}$$

$$N = \kappa x.$$

The Rindler Horizon is a Parity Horizon!

- Despite the fact that the Rindler horizon is observer dependent, it is still represented as a parity horizon in shape dynamics.
- This can be seen by noting that $N(x=0)=0$ and under the parity $x \to -x$, the lapse and spatial metric transform as:

Lapse:

$$N(x) = \kappa x \to -\kappa x = -N(x)$$

The lapse *changes sign* under parity inversion.

Spatial Metric:

$$q_{ij} = \delta_{ij} \to \delta_{ij} = q_{ij}$$

The spatial metric is *invariant* under parity inversion.

Comparison of Rindler space in GR and SD

Rindler in GR

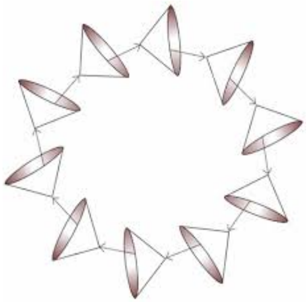
- The line element $ds^2 = -\kappa^2 x^2 dt^2 + dx^2 + dy^2 + dz^2$ is valid only for $x > 0$.
- At $x = 0$ the determinant of the space-time metric $det(g) = -\kappa^2 x^2$ goes to zero, and the horizon is a coordinate singularity.
- In order to cover the whole space one needs to describe the right $(x > 0)$ and left $(x < 0)$ Rindler wedges separately.

Rindler in SD

- The spatial metric $q_{ij} = \delta_{ij}$ is valid for all real values of x.
- The determinant of the spatial metric $det(q) = 1$ is constant and there is no singularity at the horizon.
- This is typical of parity horizons on which the lapse vanishes.
- Next we will consider a class of solutions of SD which have parity horizons on which the lapse diverges...

Chronology Protection in Shape Dynamics

The remainder of this talk will focus on physical differences between GR and shape dynamics that arise as a result of Cauchy horizons containing (on the GR side) closed time-like curves.



We use a simple example to demonstrate that where GR would predict CTCs, shape dynamics does not.

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- The simplest solutions of general relativity with CTCs are axisymmetric and have a surface on which $q_{\phi\phi} = 0$ that separates spatial infinity from some interior, acausal region.
- In the interior region, $q_{\phi\phi} < 0$ so that $\phi^a = \delta^a_\phi$ becomes timelike: $\phi^a \phi^b q_{ab} = q_{\phi\phi} < 0$.
- Since ϕ is periodic, the integral curves of ϕ^a are closed $\implies$ the interior region contains CTCs.

The Bonner Space-Time

A Simple Example: The Bonner Space-Time.

$$ds^2 = -dt^2 + 2Kd\phi dt + e^{2\Psi}\left(dr^2 + r^2 d\theta^2\right) + \left(r^2\sin^2\theta - K^2\right)d\phi^2.$$

Where $K(r,\theta) = \frac{2h}{r}\sin^2\theta$, and $\Psi(r,\theta) = \frac{h^2}{4}r^{-4}\sin^2\theta\left(\sin^2\theta - 8\cos^2\theta\right)$, and h is an area parameter.

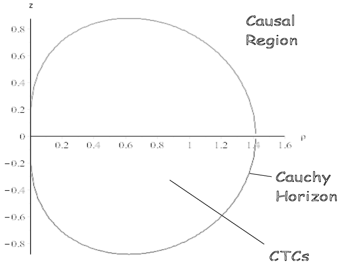
This rigidly rotating dust solution is stationary, axisymmetric and has a compact Cauchy horizon defined by:

$$r^2\sin^2\theta - \frac{4h^2}{r^2}\sin^4\theta = g_{\phi\phi} = 0 \quad \text{or} \quad r^2 = 2h\sin\theta$$

Within which there are closed time-like curves.

Shape of the Cauchy Horizon in the Bonner Space-Time

The Cauchy horizon in the Bonner Space-time has roughly the shape of a torus whose inner radius is shrunk to zero.



Above: Plot of the Bonner space-time's Cauchy horizon in the ρ-z half-plane.

The Shape Dynamic Alternative

It can be shown by the same methods just used that the shape dynamic solution that agrees with the Bonner space-time outside the Cauchy horizon is given by:

$$ds^2 = -dt^2 + 2K(u,v)d\phi dt + \frac{u^{2/3}v^{2/3}}{32}\frac{\left(v^{2/3}-u^{2/3}\right)^2}{u^{2/3}+v^{2/3}}d\phi^2$$
$$+\, e^{2\Psi(u,v)}\left[\mathbb{Q}_+(u,v)\left(u^{-2/3}du^2 + v^{-2/3}dv^2\right) + 2\mathbb{Q}_-(u,v)u^{-1/3}v^{-1/3}dudv\right]$$

Where (u,v) are coordinates adapted to the Cauchy horizon located at $u=0$, $\alpha = h^{1/3}$ and

$$\mathbb{Q}_\pm(u,v) = \frac{1}{18}\left[\frac{\alpha}{u^{2/3}+v^{2/3}} \pm \left(u^{2/3}+v^{2/3}\right)\frac{\left(v^{2/3}-u^{2/3}\right)^2}{\left(v^{2/3}-u^{2/3}\right)^2 - 16\alpha^2}\right].$$

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- The key feature is that the solution is invariant under the parity $u \to -u$.
- This transformation maps the interior into the exterior and maps the horizon into itself.
- This makes it obvious that the interior region contains no closed time-like curves.
- Similar arguments can (presumably) be made for any stationary, axisymmetric solution with a compact Cauchy horizon containing closed time-like curves.

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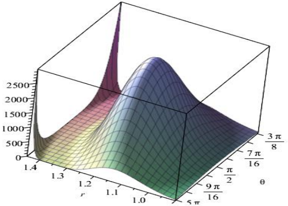
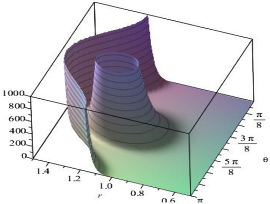
We consider the square of the Cotton tensor $\mathscr{C}^{ijk}\mathscr{C}_{ijk}$, where

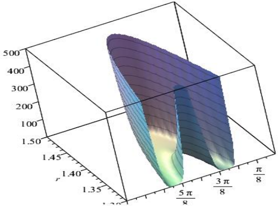
$$\mathscr{C}_{ijk} := \nabla_k \left(R_{ij} - \frac{R}{4} q_{ij} \right) - \nabla_j \left(R_{ik} - \frac{R}{4} q_{ik} \right)$$

The Cotton tensor contains all of the local information on the conformal structure of a three dimensional Riemmanian manifold.

Denominator of $\mathscr{C}^{ijk}\mathscr{C}_{ijk}$:







- Three plots of the denominator of the invariant $\mathscr{C}^{ijk}\mathscr{C}_{ijk}$ with different domains and ranges.
- Each plot displays the denominator of the invariant tending to zero on the horizon, indicating that the invariant diverges there.

A (Somewhat) More General Analysis

The spatial line element of any stationary, axisymmetric solution of shape dynamics may be locally written in the form:

$$d\ell^2 = \Omega(\rho, z)\left(d\rho^2 + dz^2 + \psi(\rho, z)d\phi^2\right)$$

The square of the cotton tensor for this line element can be written as:

$$\mathscr{C}^{ijk}\mathscr{C}_{ijk} = \frac{(\text{Third derivatives of } \psi\ldots)^2}{(\psi(\rho, z))^6(\Omega(\rho, z))^3}$$

Since the chronological horizon is defined by $\psi(\rho, z) = 0$, we see that this invariant generically diverges unless all of the (many) terms in the numerator conspire to cancel this divergence.

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Summary

- Shape dynamic black holes are physically different from general relativistic black holes.
- The known shape dynamic black hole solutions are free of physical singularities and exhibit (C)PT invariance.
- The simple example of the Bonner space-time hints that solutions of shape dynamics may avoid the formation of closed time-like curves in general.
- This would be a somewhat more parsimonious chronology protection mechanism than the corresponding arguments for general relativity.

Questions?

Thank You!