SD@ CONVERGENCE

A Shape Dynamics Workshop

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Typical universes and the origin of the second law

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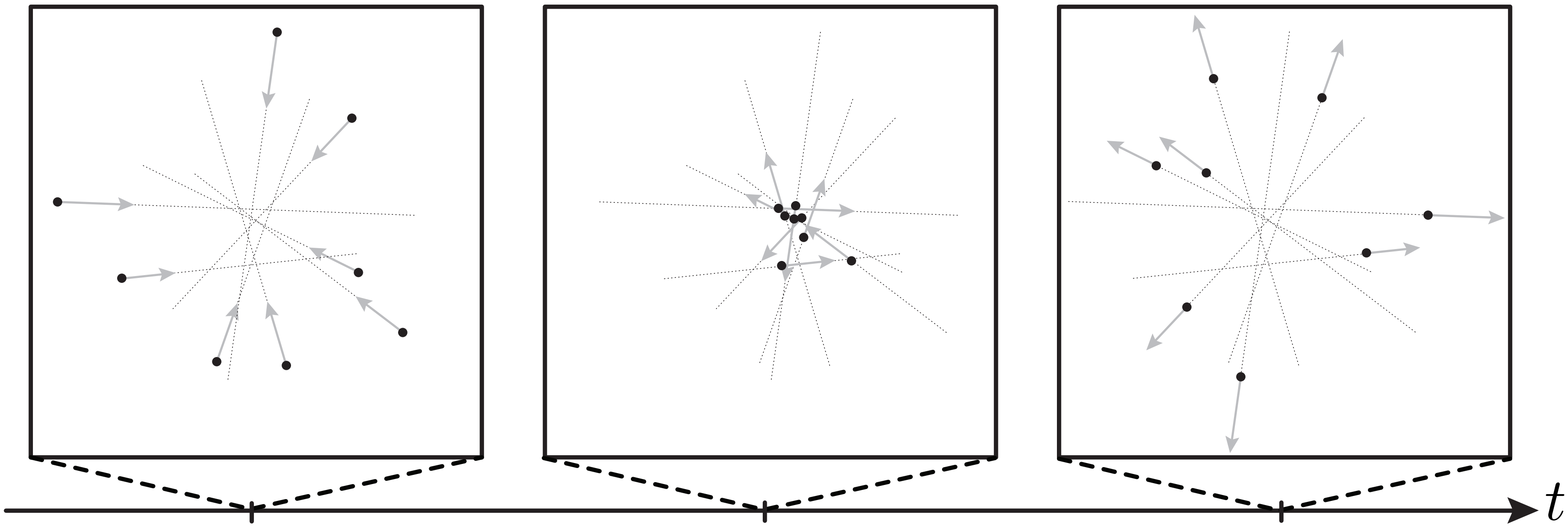
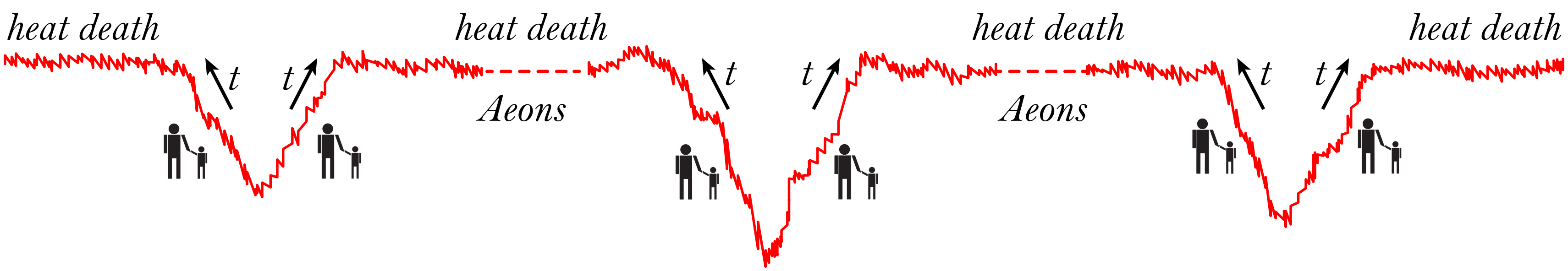
with Flavio Mercati and Tim Koslowski

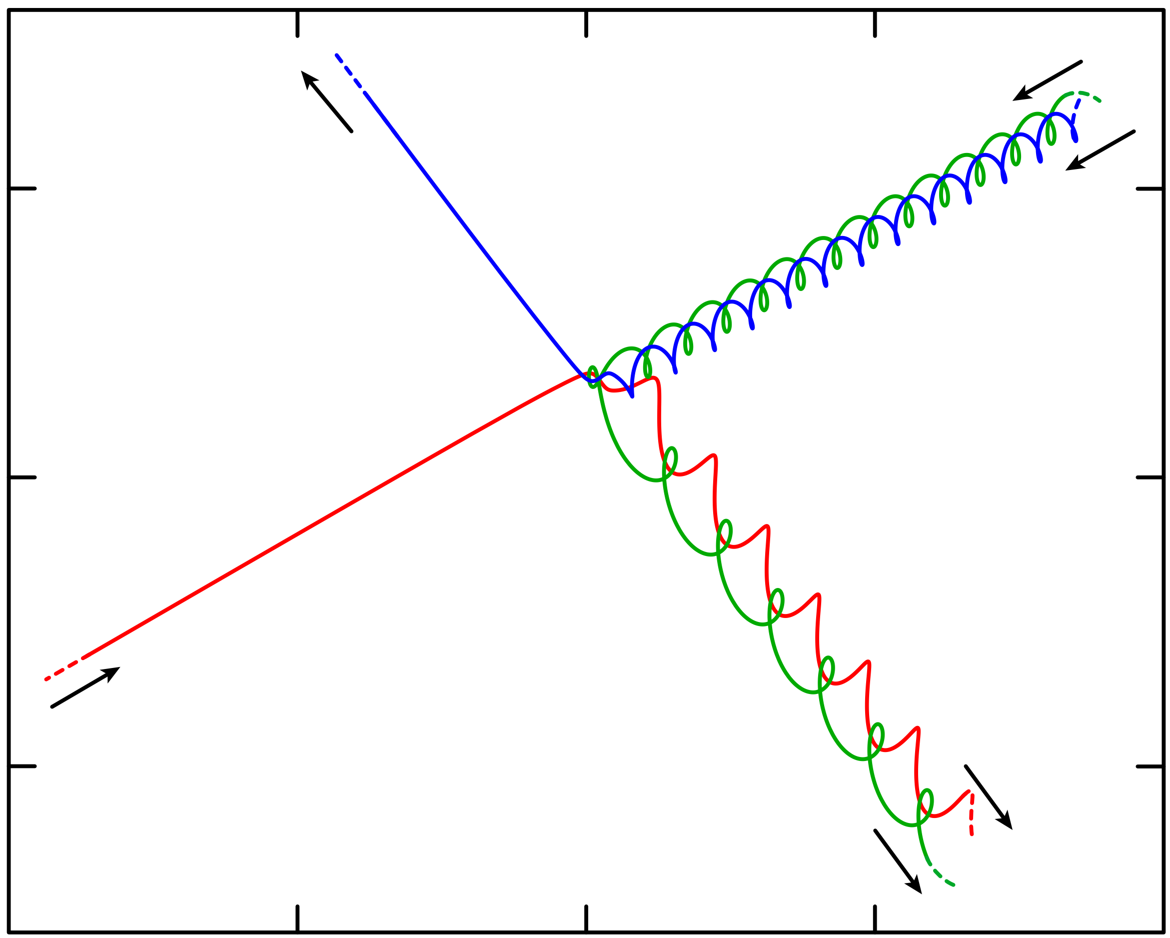
Pure Inertial Motion



Poincaré recurrences



Typical Newtonian 3-body solution



Key Concepts and Quantities

$$|V_{\text{Newton}}|/m_{\text{tot}}^2 = \frac{1}{m_{\text{tot}}^2} \sum_{a<b} m_a\, m_b\, r_{ab}^{-1} = \frac{1}{\ell} \quad \rightarrow \quad \ell = \text{‘mean harmonic length’}$$

$$I_{\text{cm}}/m_{\text{tot}} = \frac{1}{m_{\text{tot}}^2} \sum_{a<b} m_a\, m_b\, r_{ab}^{2} = L^2 \quad \rightarrow \quad L = \text{‘root mean square length’}$$

‘Complexity’ $\boxed{C_{\mathsf{S}} = \frac{L}{\ell}}$ a sensitive measure of clustering

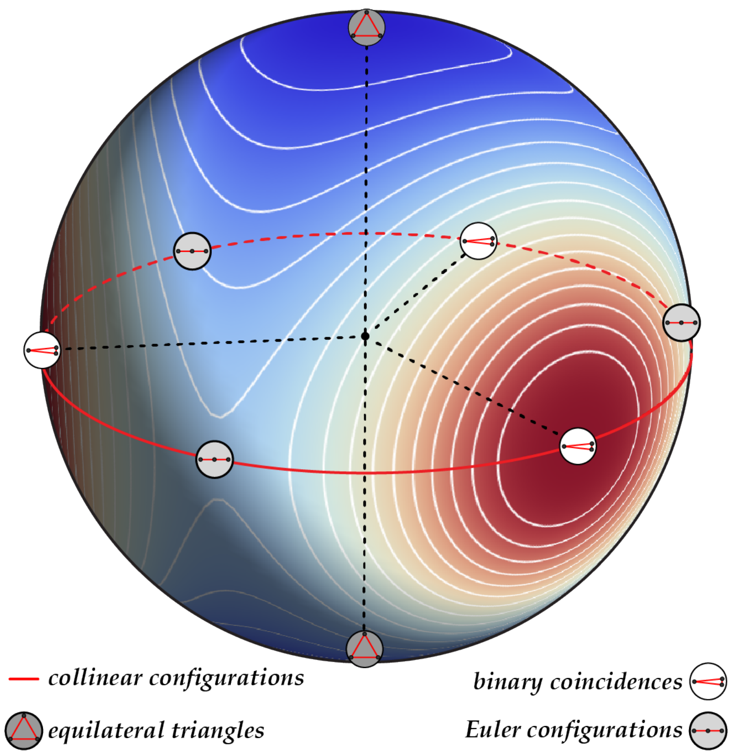
Shape Space $\mathsf{S} := \frac{\mathsf{Q}}{\mathsf{Sim}}$, Q: Newtonian configuration space

The shape-dynamical description (3-body case)

$6N - 12$ dofs. Two are **dilatational momentum** and **moment of inertia**:

$$D = \sum_{a=1}^{N} \mathbf{r}_a \cdot \mathbf{p}^a \,, \qquad\qquad I_{\text{cm}} = \sum_{a<b} m_a m_b \|\mathbf{r}_a - \mathbf{r}_b\|^2 \,,$$

What remains are the $6N - 14$ *shape* (scale-invariant) degrees of freedom, forming *shape space* and shape momenta:



If $N = 3$ shape space is the space of triangles. 2 internal angles characterize a triangle: shape space is 2D.

The Lagrange–Jacobi Relation

If $\boxed{V(\alpha \mathbf{r}_a) = \alpha^k V(\mathbf{r}_a)}$ then $\boxed{\frac{1}{2}\ddot{I}_{\text{cm}} = E_{\text{cm}} - 2(k+2)V}$ L–J or virial relation

If $\boxed{E_{\text{cm}} \geq 0}$, then since $\boxed{V_{\text{New}} < 0}$ with $\boxed{k = -1}$ we have $\boxed{\ddot{I}_{\text{cm}} > 0}$

The dilatational momentum $D = \sum_a \mathbf{r}_a \cdot \mathbf{p}^a \;\; (= \frac{1}{2}\dot{I}_{\text{cm}})$ is *monotonic*

Dynamical Similarity:

$$\mathbf{r}_a \to \alpha\, \mathbf{r}_a, \;\; t \to \alpha^{1-k/2} t$$

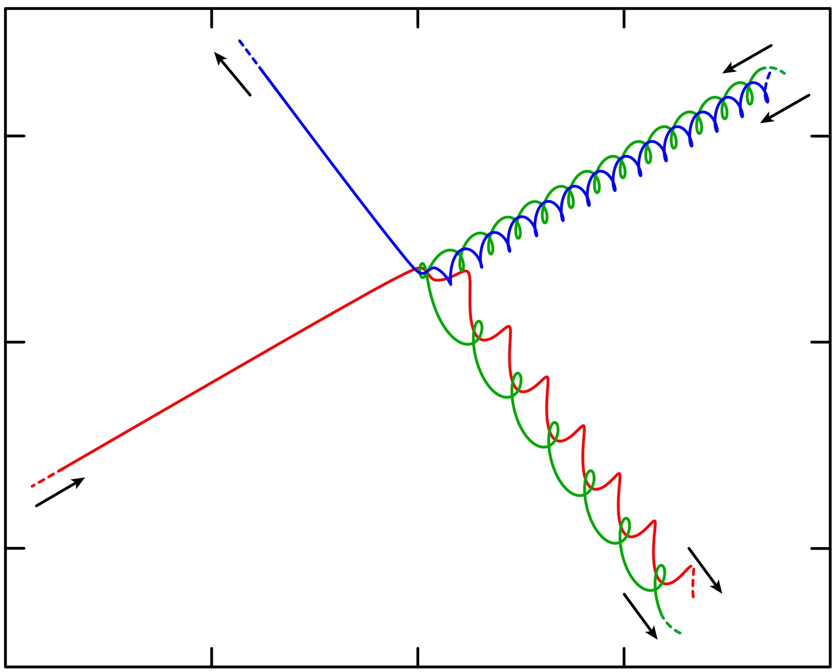
maps solutions to solutions $\Rightarrow$ absolute scale invisible on S

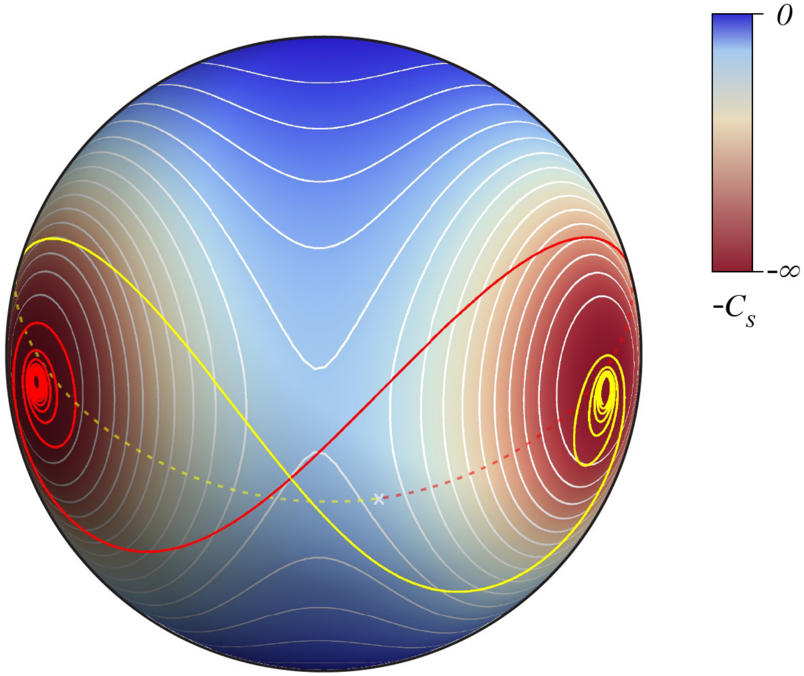
Change of scale is manifested as attractor behaviour on S

Topology of Shape Space and attractor behaviour

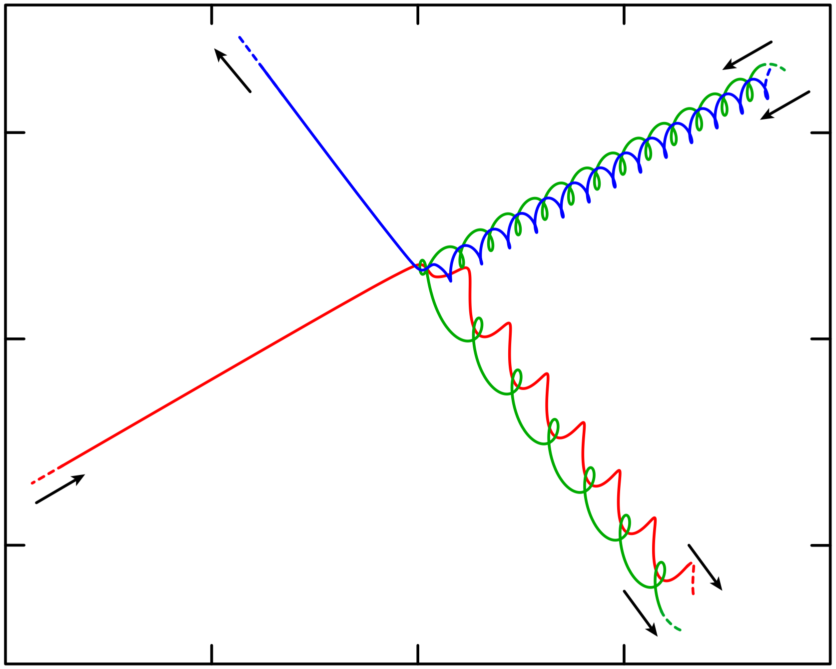
In the SD description, $\boxed{-\log C_{\mathsf{S}}}$ acts as a potential on shape space and the dynamics appears dissipative (therefore C_{S} grows secularly)

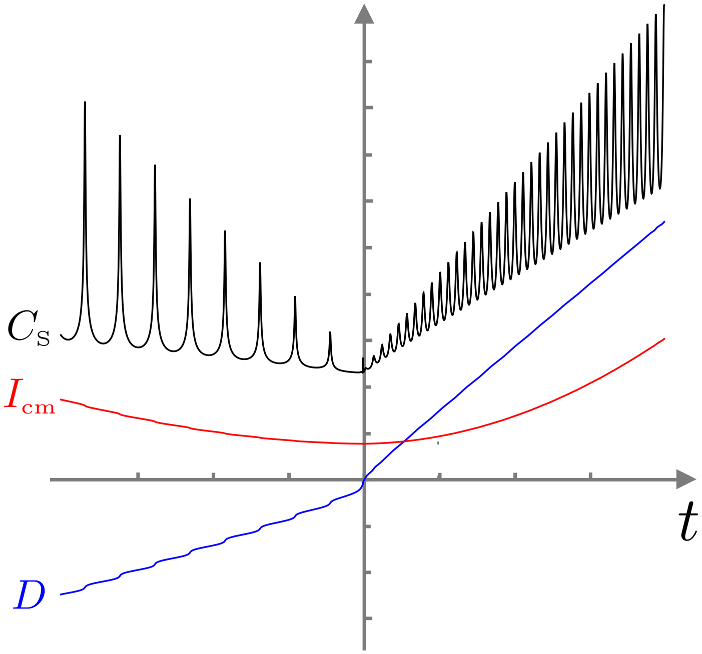
Typical 3-body solution



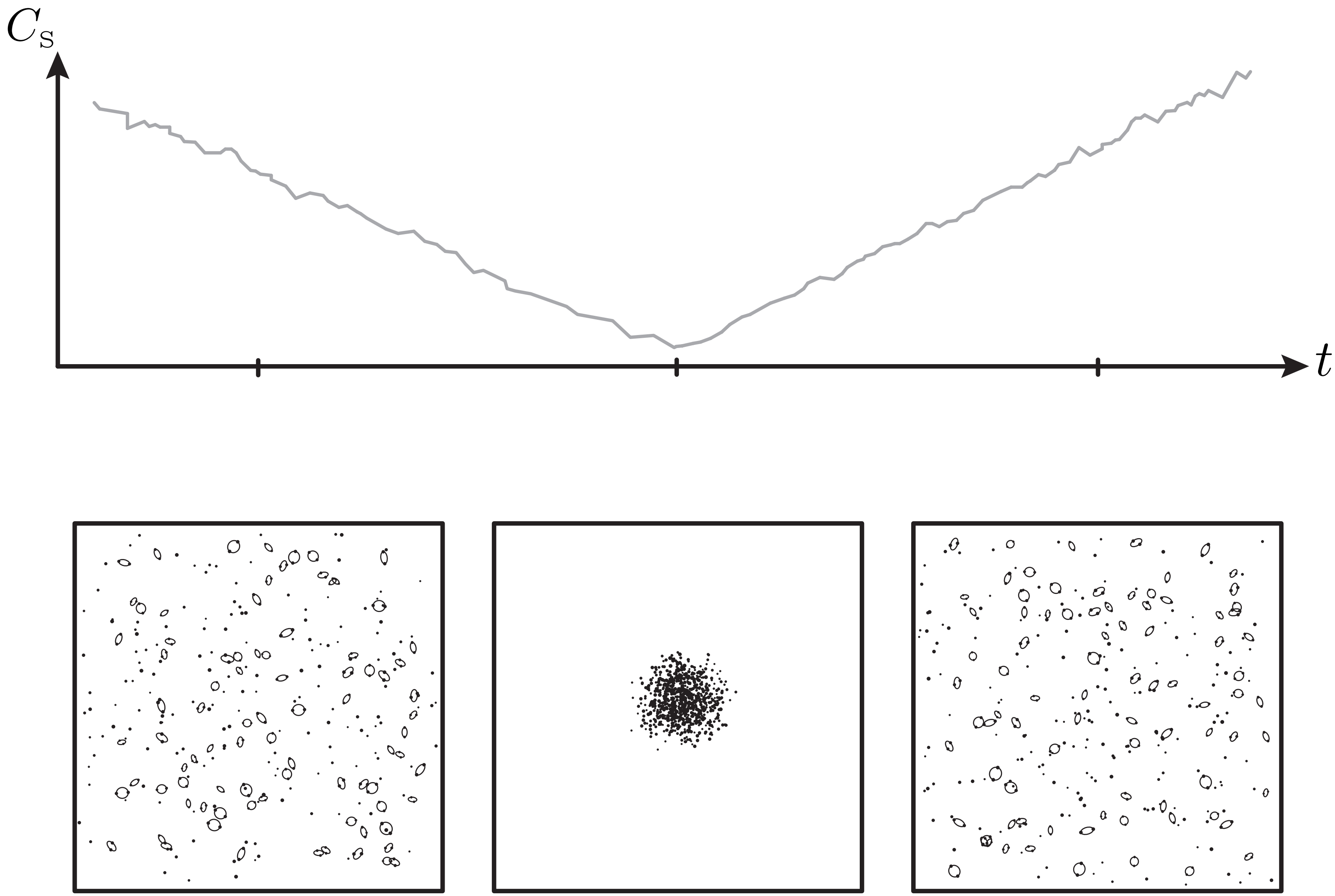


Typical 3-body solution





1000-body simulation



Theory of mid-point data

$D = 0$ **unique point** in each solution.

Natural place to set *mid-point* data.

A point and a *direction* in S determine a solution starting at $D = 0$

(an element of $\mathsf{PT}^*\mathsf{S}$, the *projectivized cotangent bundle*).

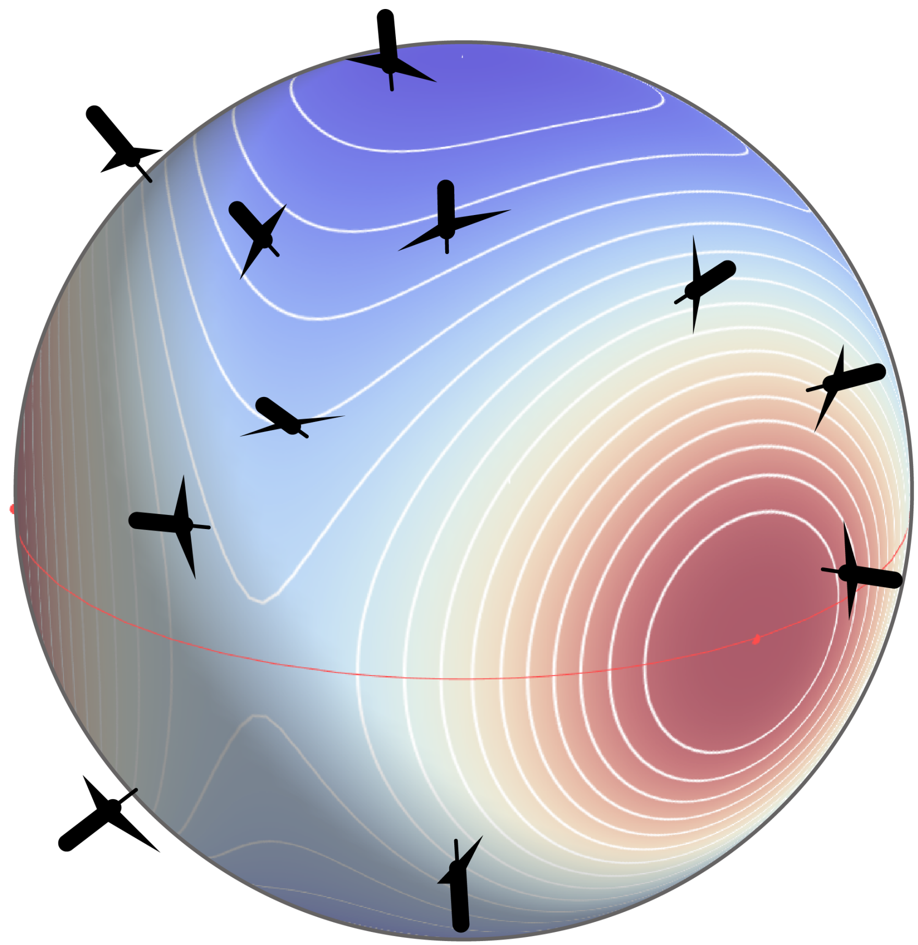
Natural induced measure on $\mathsf{PT}^*\mathsf{S}$
from symplectic structure in extended phase space.

Laplace's *principle of indifference*

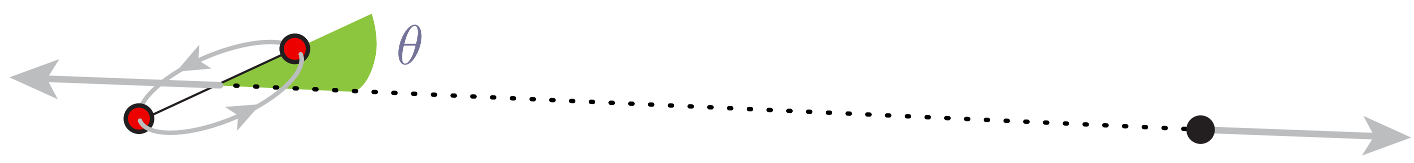
Given N possibilities and no further information, assign equal probability to each possibility.

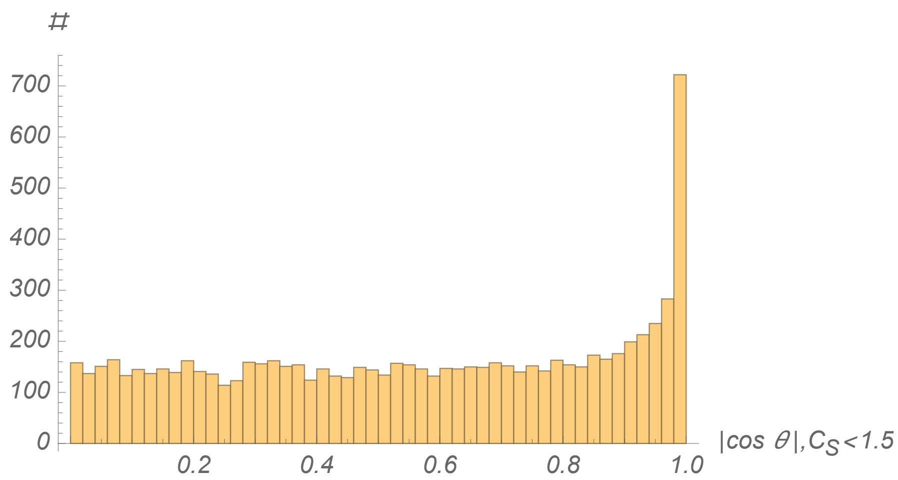
We know the law, now we want to predict what the *typical solutions* will be like.

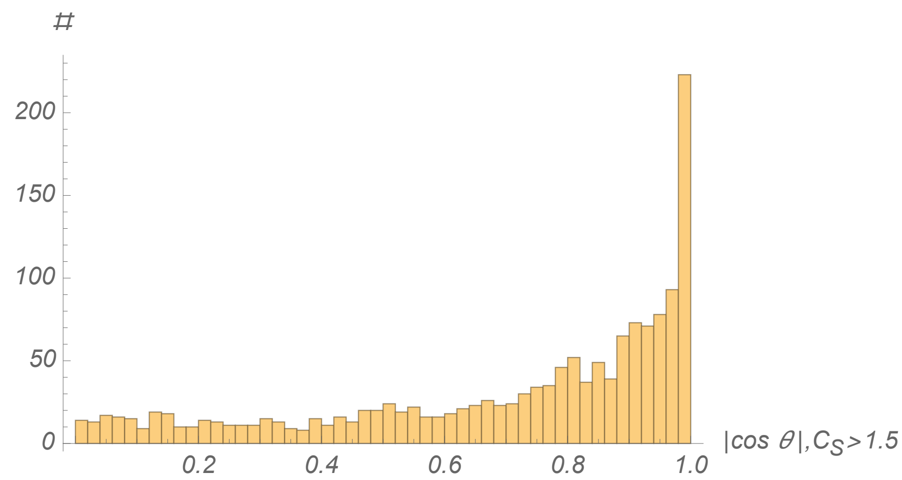
‘Blindfolded Creator’ throwing darts on S



Late-time θ vs. Janus-point C_S in 3-body problem







Complexity vs. shape space volume