1. 1st Day

1.1. **Classical BV Formalism.**

Question 1. *Let X be a smooth manifold and $S\colon X \to \mathbb{R}$ be a smooth function. Show that the derived critical locus $dCrit(S)$ of S is isomorphic to $(\mathcal{O}(T^*[-1]X), \iota_{dS})$. Namely, show*

$$\mathcal{O}(\mathit{Graph}(dS)) \otimes^{\mathbb{L}}_{\mathcal{O}(T^*X)} \mathcal{O}(X) \simeq (\mathrm{Sym}_{\mathcal{O}(X)}(T_X[1]), \iota_{dS})$$

by finding a resolution of $\mathcal{O}(\mathit{Graph}(dS))$ over $\mathcal{O}(T^*X)$.

Question 2. *Let V be an n-dimensional vector space. Let us write $\mathcal{O}(T^*[-1]V) = \mathbb{C}[x^1, \cdots, x^n, \xi_1, \cdots, \xi_n]$ with cohomological degrees $|x^i| = 0$ and $|\xi_i| = -1$ for $1 \le i \le n$. We think of $\xi_i = \partial_{x^i}$ and we have a symplectic form $\omega = \sum_{i=1}^n dx^i \wedge d\xi_i$ of cohomological degree -1. Show that Schouten–Nijenhuis bracket $\{-,-\}$ on $\mathrm{PV}(V) = \mathcal{O}(T^*[-1]V)$ is in this coordinates $\{\alpha, \beta\} = \sum_{i=1}^n \left(\frac{\partial\alpha}{\partial x^i}\frac{\partial\beta}{\partial\xi_i} - \frac{\partial\alpha}{\partial\xi_i}\frac{\partial\beta}{\partial x^i}\right)$, justifying the Poisson bracket notation.*

1.2. **Comm–Lie Koszul Duality.**

The next two questions develop some mathematical language to work with for perturbative field theory. Koszul duality asserts that roughly speaking a functor from L_∞-algebras to augmented commutative differential graded algebras (CDGA) is an equivalence. It sends an L_∞-algebra $\mathfrak{g}$ to the augmented commutative DG algebra $(C^\bullet(\mathfrak{g}), d_{\mathrm{CE}})$.

Question 3. *Show that the differential d_{CE} corresponds to an L_∞-algebra structure on $\mathfrak{g}$. [Hint: d_{CE} is a derivation so it is enough to look at its restriction to $\mathfrak{g}^*[-1]$. One may use this to make a definition of an L_∞-algebra.]*

Question 4. *Show that a map of L_∞-algebras $\mathfrak{g}_1 \to \mathfrak{g}_2$ gives a map of augmented CDGAs $C^\bullet(\mathfrak{g}_2) \to C^\bullet(\mathfrak{g}_1)$. This is in turn equivalent to a solution to the Maurer–Cartan equation in the L_∞-algebra $C^\bullet_{red}(\mathfrak{g}_1) \otimes \mathfrak{g}_2$. Here the reduced cochain $C^\bullet_{red}(\mathfrak{g}_1)$ is defined to be the kernel of the natural augmentation map.*