1. Consider a cohomological 2d QFT equipped with two left boundary conditions A and B. Their endomorphisms consist of the identity only.

$$\mathrm{End}(A) = \mathrm{End}(B) = \mathbb{C} \tag{1}$$

Soppose that $\mathrm{Hom}(B, A)$ is empty but $\mathrm{Hom}(A, B)$ is some vector space V in ghost number (degree) 0.

(a) If $V = \mathbb{C}$, which other boundary conditions can you obtain from solutions of Maurer-Cartan equations/complexes built from A and B?

Can you define a 2π rotation/Serre functor $\mathcal{S}$, so that $\mathrm{Hom}(A, B) = \mathrm{Hom}(B, \mathcal{S} \circ A)^*$?

If so, you should have obtained a full description of the boundary conditions in the A-twisted LG theory with $W = \phi^3$ superpotential.

(b) If $V = \mathbb{C}^2$, which other boundary conditions can you obtain from solutions of Maurer-Cartan equations/complexes built from A and B?

Can you define a 2π rotation/Serre functor $\mathcal{S}$, so that $\mathrm{Hom}(A, B) = \mathrm{Hom}(B, \mathcal{S} \circ A)^*$?

If so, you should have obtained a full description of the boundary conditions in the B-twisted $\mathbb{C}P^1$ sigma model

(c) What about $V = \mathbb{C}^3$? Can you define a Serre functor? Can you enlarge the category further so that it admits a Serre functor?