2. EXERCISE: DAY 2

(1) Consider the following isomorphism of $D = 2N = (2,2)$ SUSY algebra which switches

$$Q_+ \leftrightarrow \bar{Q}_+$$

and keep all other operators. Show that under the isomorphism, topological A-twist and topological B-twist are switched. This is the origin of mirror symmetry.

(2) Let Φ be a chiral superfield, written into component

$$\Phi = \phi(y^\pm) + \theta^\pm \psi_\pm(y^\pm) + \theta^+\theta^- F(y^\pm), \quad y^\pm = x^\pm + i\bar{\theta}^\pm\theta^\pm.$$

Show that the SUSY transformation $\Phi \to \delta\Phi$ where $\delta = \epsilon^\pm Q_\pm + \bar{\epsilon}^\pm \bar{Q}_\pm$, is read in components by

$$\begin{cases} \delta\phi = \epsilon^+\psi_+ + \epsilon^-\psi_- \\ \delta\psi_\pm = 2i\bar{\epsilon}^\pm\partial_\pm\phi \pm \epsilon^\mp F \\ \delta F = -2i\bar{\epsilon}^+\partial_+\psi_- + 2i\bar{\epsilon}^-\partial_-\psi_+ \end{cases}$$

(3) Let Φ^i be chiral superfields on $D = 2, N = (2,2)$ superspace. In components, $\Phi^i = (\phi^i, \psi^i_\pm, F^i)$. Justify that SUSY invariant action

$$S_D = \int d^2x \int d^2\theta d^2\bar{\theta} K(\Phi^i, \bar{\Phi}^i)$$

takes the following form in components

$$S_D = \int d^2x \left(-\frac{1}{2} g_{i\bar{j}}\partial^\mu\phi^i\partial_\mu\bar{\phi}^{\bar{j}} + i g_{i\bar{j}}\bar{\psi}^{\bar{j}}_- D_+\psi^i_- + i g_{i\bar{j}}\bar{\psi}^{\bar{j}}_+ D_-\psi^i_+ + R_{i\bar{j}k\bar{l}}\psi^i_+\psi^k_-\bar{\psi}^{\bar{j}}_-\bar{\psi}^{\bar{l}}_+ + g_{i\bar{j}}(F^i - \Gamma^i_{jk}\psi^j_+\psi^k_-)(\bar{F}^{\bar{j}} - \Gamma^{\bar{j}}_{\bar{k}\bar{l}}\bar{\psi}^{\bar{k}}_-\bar{\psi}^{\bar{l}}_+) \right)$$

Here K is the Kahler potential, $g_{i\bar{j}} = \partial_i\partial_{\bar{j}}K$ is the Kahler metric, and $R_{i\bar{j}k\bar{l}}$ is the curvature tensor. The following observation may be helpful for this computation

$$\int d^2x \int d^2\theta d^2\bar{\theta} K(\Phi^i, \bar{\Phi}^i) = \int d^2x D_+D_-\bar{D}_+\bar{D}_-K|_{\theta=\bar{\theta}=0}.$$

$$\phi^i = \Phi^i|_{\theta=\bar{\theta}=0}, \quad \psi^i_\pm = D_\pm\Phi^i|_{\theta=\bar{\theta}=0}, \quad F^i = D_+D_-\Phi^i|_{\theta=\bar{\theta}=0}$$

Generalize this to compute the F-term of SUSY invariant action

$$\int d^2x \int d^2\theta W(\Phi) + c.c.$$

where $W(z^i)$ is a holomorphic function.

(4) Show that in the A-twist, the Q_A-cohomology of local observables on the worldsheet is given by the de Rham cohomology $H^\bullet_{dR}(X)$ of the target. Similarly, in the B-twist, the Q_B-cohomology of local observables on the worldsheet is given by the Dolbeault cohomology $H^\bullet(X, \wedge^\bullet T_X)$. In this computation, you will need the formula of SUSY transformation above.