3 Lecture 3

1. Consider the local model of a supersymmetric quantum mechanics with $X = \mathbb{C}$ and Morse function $h = \omega|z|^2$. The supercharges are

$$Q = (+\partial_{\bar{z}} + m\omega)d\bar{z}$$
$$Q^\dagger = (-\partial_z + m\omega)i_{\partial/\partial\bar{z}}$$

   (a) Show that the normalisable supersymmetric ground states are

$$\Psi^{(n)} = \begin{cases} C_n e^{-\omega|z|^2} z^n & \omega > 0 \\ C'_n e^{\omega|z|^2} \bar{z}^n d\bar{z} & \omega < 0 \end{cases}$$

   for some constants C_n, C'_n.

   (b) Show that the normalised wavefunctions

$$C_n = \sqrt{\frac{(2\omega)^{n+1}}{n!\pi}} \qquad C'_n = \sqrt{\frac{(-2\omega)^{n+1}}{n!\pi}}$$

   are orthonormal for both $\omega > 0$ and $\omega < 0$.

   (c) Now define normalised "in" states by

$$\Psi^{(n)}_{\text{in}} = \begin{cases} \sqrt{\frac{n!\pi}{(2\omega)^{n+1}}} e^{\omega|z|^2} \Psi^{(n)} & \omega > 0 \\ \sqrt{\frac{(-2\omega)^{n+1}}{n!\pi}} e^{\omega|z|^2} \Psi^{(n)} & \omega < 0 \end{cases}$$

   Show that in the limit $|\omega| \to \infty$,

$$\Psi^{(n)}_{\text{in}} \to \begin{cases} z^n & \omega > 0 \\ \frac{(-1)^n}{n!} \partial_z^n \delta(z, \bar{z}) d\bar{z} & \omega < 0 \end{cases}$$

   (d) Explain why the wavefunctions for $\omega < 0$ can be expressed in the form $\bar{\partial}\frac{1}{z^{n+1}}$.